A RATIONAL NORMAL FORM FOR CERTAIN QUARTICS*

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A quartic equation with rational coefficients, irreducible in the field R of rational numbers, will have for its Galois group (for the field R) one of the following:

I. The symmetric group of order 24.

II. The alternating group of order 12.

III. A group of order 8 such as the one whose substitutions are 1, (12), (34), (13)(24), (12)(34), (14)(23), (1324), (1423).

IV. A cyclic group of order 4 such as 1, (1324), (12)(34), (1423).

V. The 4-group 1, (12)(34), (13)(24), (14)(23).

It is the purpose of this note to give what seems to be a more direct proof of a theorem of Wiman:†

THEOREM. *Any rational quartic, irreducible in R, whose Galois group for R is either* III, IV, *or* V *above, can be transformed by a rational Tschirnhausen transformation into the form $y^4+py^2+q=0$, where p and q are rational.*

It will then follow as a corollary that p and q can be made integral; this can be accomplished by a simple additional transformation.

We may consider the given quartic in the reduced form

$$x^4 + a_2x^2 + a_3x + a_4 = 0, \quad (a_1 = s_1 = 0,\, a_3 \neq 0). \tag{1}$$

Apply the quadratic transformation

$$y = x^2 + k_1x + k_2. \tag{2}$$

Now the transformed equation will lack its second and fourth terms provided $\sum y=0$, $\sum y^3=0$.‡ The first of these equa-

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‡ The summations extend over the 4 roots of (1).