

## REVISION OF PASCH'S GEOMETRY

*Vorlesungen über neuere Geometrie.* By Moritz Pasch. Second edition. With an appendix: *Die Grundlegung der Geometrie in historischer Entwicklung* by Max Dehn. (Die Grundlehren der mathematischen Wissenschaften, Band 23.) Berlin, Springer, 1926. x+275 pp.

In 1882, Pasch's *Vorlesungen über neuere Geometrie* inaugurated a new era in the development of projective geometry. For the first time, projective geometry was made a purely deductive science founded upon and logically developed from a complete system of postulates. At the same time, since the postulates completely ignored the concept of parallelism, it was definitely made independent of euclidean or any metric geometry. This was in accord with a previous discovery by Klein that the theorems of projective geometry are independent of the parallel postulate.

The book was reprinted in 1912 with a short appendix and in 1913 it was translated into Spanish and the appendix of the 1912 edition was distributed over the book as smaller appendices at the ends of different sections.

The second edition just published follows the plan of the Spanish edition. The portion revised is not extensive. The rewritten material totals about two pages in the second edition and takes the place of about five pages of the first edition. It consists almost entirely of simplified proofs. In one place (page 47, second paragraph) a proof that had covered two pages is now given in one short paragraph.

The second edition contains a total of eight pages of added material. This is inserted piecemeal as supplements to the various sections or as isolated paragraphs between two formerly adjacent paragraphs. Some new theorems are added, for example: p. 7, Kernsatz IX; p. 25, Lehrsätze 13, 14, 15; p. 39, theorem by Ventura Reyes y Prosper; p. 76, Sätze J and P. Aside from these, the supplementary material consists mainly of comments and references.

The typographical errors of the first edition are corrected. This was not done in the reprint of 1912.

The evident desire to supplement rather than revise this important pioneer work is best explained by the author himself in his preface to the second edition. After discussing some of the more recent methods and results, he continues, "I do not desire to incorporate these new geometric concepts in the new edition, but prefer to keep the point of view of the first edition. If the reader will allow himself to be carried back in imagination for half a century, he may wish at least to acquaint himself with the geometry of that period and with the position this book had in its time in the foundations of geometry."

As the preferable alternative to enlarging and revising his own book so as to include the new concepts, Pasch requested Dehn to write an appendix covering this field. The last eighty-seven pages of the book