THE FUNCTIONAL EQUATION DEFINING DIOPHANTINE AUTOMORPHISMS*

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1. *Introduction*. A form $f(x_1, \cdots, x_m)$ is said to admit a *diophantine automorphism* if there exist m forms

$$\phi_1(x_1, \cdots, x_m), \cdots, \phi_m(x_1, \cdots, x_m),$$

such that

$$(1) \qquad f[\phi_1(x_1, \cdots, x_m), \cdots, \phi_m(x_1, \cdots, x_m)] = f^n(x_1, \cdots, x_m),$$

where n is a positive integer greater than 1, the coefficients of the forms being integers or rational integral functions of one or more parameters with integral coefficients. In view of Bell's remarks (*A diophantine automorphism*, this Bulletin, vol. 28, p. 71) on the scarcity of what he calls *proper* diophantine automorphisms, it seems desirable to seek solutions of (1), regarded as a functional equation, no restrictions being placed on the algebraic nature of the solution or the coefficients.

Three methods of obtaining solutions of this functional equation are offered in the present paper. The first method gives a solution corresponding to any one-parameter continuous group having certain properties. The solution is generally a transcendental function, but I present this method here as the functional equation is of interest in analysis. In the second method f is assumed known, and the ϕ's are to be determined. It is shown that if there exists a Cremona transformation of which $x_i' = f(x_1, \cdots, x_m)$ is a constituent, then there exist *rational* functions $\phi_1, \cdots, \phi_m$ satisfying (1). This leads to rational solutions of the diophantine equation

$$f(x_1, \cdots, x_m) = u^n.$$

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