CONCERNING CONNECTED AND REGULAR POINT SETS*

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In this paper an extension will be given of a theorem of the author's† which states that if A and B are any two points of a continuous curve M, and K denotes the set of all those points of M which separate‡ A and B in M, then $K+A+B$ is a closed set of points. It will be shown that if A and B are any two points of any connected and regular (connected im kleinen) point set M, and K denotes the set of all those points of M which separate A and B in M, then $K+A+B$ is a closed and bounded set of points. This extended theorem is applied to show that a simple continuous arc may be defined as a connected and regular point set which is irreducibly connected§ between some two of its points.

THEOREM 1. *If A and B are any two points of a connected and regular point set M, and K denotes the set of all those points of M which separate A and B in M, then $K+A+B$ is a closed and bounded set of points.*

PROOF. I shall first show that $K+A+B$ is closed. Suppose, on the contrary, that there exists a point P which does not belong to $K+A+B$ but which is a limit point of K. Then there exists a sequence of points $S=Y_1, Y_2, Y_3, \cdots,$

* Presented to the Society, San Francisco Section, June 18, 1927.

† G. T. Whyburn, *Some properties of continuous curves*, this Bulletin, vol. 33 (1927), pp. 305–308.

‡ The points A and B of the connected point set M are said to be separated in M by the point X of M provided that $M-X$ is the sum of two mutually separated point sets containing A and B respectively.

§ A connected point set M is said to be irreducibly connected between two of its points A and B provided that no proper connected subset of M contains both A and B. See N. J. Lennes, American Journal of Mathematics, vol. 33 (1911), p. 308. See also Knaster and Kuratowski, *Sur les ensembles connexes*, Fundamenta Mathematicae, vol. 2 (1921), p. 206.