THE PROBABILITY LAW FOR THE INTENSITY OF A TRIAL PERIOD, WITH DATA SUBJECT TO THE GAUSSIAN LAW*

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1. *Introduction.* Making use of Lord Rayleigh's† probability law for the resultant of n vibrations of fixed and equal amplitude but of chance phase, Schuster‡ found a probability law for the ratio of the square-root of the intensity of a trial period to the constant term of the Fourier development, and suggested the use of this as a criterion for the possible fortuitous nature of results apparently supporting the existence of periods under investigation. However, as periodicities in a sequence may be invariant to a change of origin, it seems desirable to have a criterion based upon intensity alone, which is thus invariant. Such a criterion will be derived in this paper. Some criterion—as Schuster pointed out—is indispensable in periodogram analysis. It signifies little that one trial period is more probable than another if all the apparent periodicities could be easily the result of chance. The question, then, is this : What fluctuations in the intensity may be expected when the data are subject to chance,—or, more specifically, to the Gaussian law?

2. *The Intensity of a Period and its Probability Law.* Suppose that the probability $p(x)$ that X_r will take on a value $\leqq x$ is given by

$$(1) \quad p(x) = \frac{h}{\pi^{1/2}} \int_{-\infty}^{x} e^{-h^2(t-b)^2} dt, \quad (r = 1, 2, \cdots, km\ ;\ km = n),$$

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† *On the resultant of a large number of vibrations of the same pitch and of arbitrary phase*, Philosophical Magazine, (5), vol. 10 (1880), pp. 73–78.

‡ *On lunar and solar periodicities of earthquakes*, Proceedings of the Royal Society of London, vol. 61 (1897), pp. 455–465.