ON THE POLYNOMIAL CONVERGENTS OF POWER SERIES*

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In this paper we will consider the power series

$$\sum_{i=0}^{i=\infty} a_i z^i$$

with a unit radius of convergence. M. B. Porter† defined a set of convergents for this series in such a way that the set had many interesting properties.‡ He proved that at least one point of the unit circle was a limit point of the zeros of these convergents, and that in the neighborhood of every point of this circle some of the convergents took on values whose moduli were arbitrarily small. Jentzsch§ has shown that every point of the unit circle is a limit point of the zeros of the complete set $\{f_n(z)\}$ of convergents defined by

$$f_n(z) = \sum_{i=0}^{i=n} a_i z^i$$

for all positive integral values of n. It is the purpose of the present paper to show that the set of convergents defined by Porter has every point of the unit circle as a limit point of its zeros. As a special case of this result we get the theorem of Jentzsch. Other properties of these convergents are developed also.

Let $f(z)$ be the analytic function defined by the above power series inside of the circle $|z|=1$ and let z_0 be a point

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† Annals of Mathematics, vol. 8 (1906–7), pp. 189–192.

‡ Of extreme importance is the fact that Porter embodied in this set of convergents all of the properties that were later assigned by Montel (see his *Leçons sur les Séries de Polynomes etc.*, Paris, 1910) to his *normal families* of functions.

§ Acta Mathematica, vol. 41 (1917), pp. 253–270.