SHORTER NOTICES

A Course of Differential Geometry. By J. E. Campbell. Oxford, Oxford University Press, 1926. xv+261 pp.

The manuscript for this book was completed by Professor Campbell shortly before his death in 1924. It was prepared for the press by Professor E. B. Elliott. The book was evidently planned to prepare the reader to understand differential geometry as applied in modern relativity theory. The methods of tensor analysis have been employed throughout.

The first chapter gives a brief but quite satisfactory introduction to the tensor theory associated with a symmetric quadratic differential form with n independent variables. The Christoffel symbols of three and four indices are defined and discussed and the operation of covariant differentiation is introduced without being so called. At the end of this chapter an Einstein space is defined as one in which certain tensor components vanish but no explanation of the definition is given at this point.

The next nine chapters contain a treatment of differential geometry of ordinary space, tensor methods being used almost exclusively. Particular attention is given to the following subjects: equivalence of forms, geodesics, curvature deformation of surfaces, congruences, curves in space and on a surface, ruled surfaces, minimal surfaces, conformal representation, orthogonal surfaces.

In order to understand these nine chapters the reader must have previously acquired the main facts of differential geometry. Little explanation outside of the analysis is offered and the geometrical concepts are usually introduced without definition. The purposes of this part of the book undoubtedly are to show the power of vector methods and to prepare the way for extensions to hyperspace.

In the last four chapters we return to the consideration of the quadratic form with n independent variables and are thus led to geometry in n-way space. As a generalization of the straight line in the plane we have the geodesic. Gauss's measure of curvature has as a natural extension Riemann's measure of curvature, in which a tangential n-fold takes the place of the tangent plane.

The fundamental form for an $(n+1)$-way space may be taken as

$$\phi du^2 + b_{ik}\, dx_i\, dx_k \qquad (i, k = 1, \cdots, n),$$

where ϕ, b_{ik} are functions of $x_1, \cdots, x_n$ and u. The surface $u=0$ is an arbitrary surface in the space. This $(n+1)$-way space is an Einstein space if for all such surfaces lying in it

$$\sum \frac{1}{R_i R_k} + \frac{1}{2} A = 0,$$

where the first term represents the sum of the products, taken two at a time, of the reciprocals of the principal radii of curvature of the surface, and where