CULLIS ON MATRICES

Matrices and Determinoids. By C. E. Cullis. University of Calcutta Readership Lectures. Cambridge University Press, Vol. III, Part 1, 1925. xviii+681 pp.

The first two volumes of this treatise were reviewed in this Bulletin (vol. 26 (1920), pp. 224–233). The present volume continues the development, which has become more extensive than at first contemplated, so that the third volume has had to appear in two parts. The second part, it is stated, will deal chiefly with structural matrices. The author's avowed aim is to carry forward the program of Sylvester, who conceived a magnificent treatise on universal algebra, which he never carried out. The present treatise to a large extent is realizing this program. To Professor Cullis, matrices means practically all of algebra, and it is easy to see from his standpoint that every expression in algebra, whether it depends upon one or upon more variables, is in reality a function of some sort of matrix. To the reviewer this is tantamount to saying that every function of one or more variables is ipso facto a function of a vector of a space of the same number of dimensions, or of various vector products in such space. The difference is simply one of point of view or mode of statement. It is simpler, in the reviewer's opinion, to make the whole subject a branch of the theory of hypernumbers, than to build up a theory of "sets," although this is really following the tradition of Sir W. R. Hamilton, the creator of quaternions. The author insists rightly upon the matrix as a single entity, and considers that a *pure* calculus will be determined by a system of scalar numbers, so that he would introduce hypernumbers later, the ordinary quaternion, for instance, being a product of a scalar matrix and a matrix whose elements are the usual $1, i, j, k$. While this is logical, and is a justifiable line of development, it seems to the reviewer to complicate the subject unnecessarily. The author further considers that on the applied side, every physical entity can be represented by a matrix, and he certainly would find plenty of justification for this view in modern physics. His assertion (p. vi) that the expressions would be two-dimensional tables, hardly seems tenable, however. For witness the modern tensor theory.

The scope of this volume is best indicated by a brief indication of the contents of the various chapters. The first three (XX, XXI, XXII) deal with rational integral functions of any number of variables, irresoluble and irreducible functions and factors, common factors, resultants, eliminants, discriminants, and common roots. Symmetric functions are considered specially in Chapter XXII. In Chapter XXIII begins the consideration of rational integral functional matrices, that is, matrices whose elements are rational integral functions of scalar variables, $x, y, z, \cdots$. The definition is given and properties proved of irreducible and irresoluble divisors of such matrices. These divisors are rational integral functions of the same variables as the matrix, and are of order i when they will divide every minor determinant of order i which arises