CONCERNING THE BOUNDARIES OF DOMAINS OF A CONTINUOUS CURVE*

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We shall consider a space M consisting of all the points of a plane continuous curve M†, and all point sets mentioned are assumed to be subsets of M. A connected set of points D of M is said to be an *M-domain* if $M-D$ is closed. The set of all limit points of D which do not belong to D is called the *M-boundary* of D. If D is an M-domain, D' denotes the M-boundary of D, and $\overline{D}$ denotes the set $D+D'$. The M-boundary of an M-domain D is closed but not necessarily connected, even if D is simply connected, as we may easily show by examples. If N is a continuum, a maximal connected subset of $M-N$ is called a *complementary M-domain* of N.

THEOREM I.‡ *Every closed and connected subset of the M-boundary of a complementary M-domain of a continuous curve N is a continuous curve.*

PROOF. Let K denote a closed and connected subset of the M-boundary of a complementary M-domain D of N. Suppose K is not connected im kleinen. Then there exist§ two concentric circles C_1 and C_2 (let r_i denote the radius of C_i and let $r=r_1-r_2>0$) and an infinite sequence of subcon-

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† A point set which is closed, connected and connected im kleinen is called a *continuous curve*. In general it may be either bounded or unbounded.

‡ For the case where M is the entire plane and N is bounded, see R. L. Wilder, *Concerning continuous curves*, Fundamenta Mathematicae, vol. 7 (1925), p. 361.

§ See R. L. Moore, *Report on continuous curves from the viewpoint of analysis situs*, this Bulletin, vol. 29 (1923), p. 296. This theorem is stated by Moore for a bounded continuum, but the theorem remains true without the condition of boundedness.