ON THE GEOMETRY OF LINEAR DISPLACEMENT*

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1. *Introduction.* The first to consider the displacement of a vector to a point at an infinitesimal distance in a non-euclidean geometry seems to have been L. E. J. Brouwer for the case of space of constant curvature.† Its importance for differential geometry was pointed out for the first time, however, by Levi-Civita (1917. 1),‡ for the case of a hypersurface immersed in an euclidean space, that is, the case of an n-dimensional Riemannian displacement. It was with this paper of Levi-Civita that the new development of differential geometry was started. Weyl (1918. 1 ; 1918. 3) first pointed out the way in which Levi-Civita's results could be generalized and he originated what we call *the Weyl displacement.* Even before Weyl and Levi-Civita, generalizations of analogous character had been suggested by Hessenberg (1916. 1). The geometry of linear displacement was developed in the papers of König (1919. 1 ; 1920. 1), Eddington (1921. 1 ; 1923. 4), Weyl (1918. 1 ; 1918. 3), and Schouten (1922. 1). Schouten gave a general classification of the different geometries defined by linear displacements, which he published later in his book (1924. 9). We adopt in general Schouten's notation.

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† L. E. J. Brouwer, *Het krachtveld der nieteuklidische negatief gekromde ruimten*, Verslagen Akademie Amsterdam, vol. 15 (1906), pp. 75–94 ≡ *The force field of the non-euclidean spaces with negative curvature*, Proceedings Academy Amsterdam, vol. 9 (1906), pp. 116–133. See however also a remark of H. Poincaré in A. Dall' Acqua, *Sulla teoria delle congruenze di curve in una varietà qualunque a tre dimensioni*, Annali di Matematica, (3), vol. 6 (1901), pp. 1–40.

‡ Such references refer to the bibliography at the end of this paper (see p. 558). Independently of Levi-Civita, Schouten obtained analogous results (1918.4).