APPELL ON TENSOR CALCULUS

Traité de Mécanique Rationnelle. Vol. V: *Eléments de Calcul Tensoriel, Applications Géométrique et Mécanique.* By Paul Appell with the collaboration of René Thiry. Paris, Gauthier-Villars, 1926. vi+198pp.

This work of Appell on the absolute differential calculus and its applications forms the 5th volume of his *Traité de Mécanique Rationnelle* and is intended as the first part of a treatment of the mechanics of the theory of relativity. Yet it would seem that the present is no time for the appearance of such a text. The uncertain position of the general theory of relativity as shown by the unsuccessful attempts of Einstein and others to develop a field theory of electromagnetic phenomena, together with the new views, beginning with Heisenberg and ending with Schrödinger, which have lately been advanced, show that we do not yet stand on solid ground. It may be necessary to make substantial changes in the mathematical foundations of relativity theory. As a consequence it is better to regard this last book of Appell merely as an elementary exposition of some work in pure mathematics without regard to applications.

One feature of the book which deserves considerable adverse criticism is the lack of any *real* application to mechanics. A few of the simpler equations of mechanics have been written in the tensor notation on the basis of the three-dimensional euclidean geometry. But no applications to mechanics of the more general mathematical developments have been made.

The first chapter is devoted to certain fundamental theorems on linear and quadratic forms, and as such is introductory to the tensor calculus proper; an especially small type of printing is employed. A treatment of these fundamental theorems is usually omitted by writers on this subject. I regard this chapter as a valuable addition to the book.

Owing to the all too many books which have appeared in recent years on the theory of relativity, as well as the increasing number of mathematical papers written on the basis of the tensor calculus, there does not remain much room for originality in the second chapter which is devoted to the elements of this subject. However, on account of the importance of this chapter for the later developments, it may be well to consider it in some detail. Going out from the two fundamental requirements governing the equations of transformation of *a system of functions* (f), see §20, the system of functions known as a tensor is introduced as a simple system satisfying the fundamental requirements. After this the rules of tensor algebra are laid down, the fundamental quadratic differential form $g_{\alpha\beta}dx^\alpha dx^\beta$ is produced, and the differential equations of geodesics are developed in the ordinary manner. Then follows a cumbersome treatment of covariant differentiation of a tensor on the basis of the artificial and foreign idea of a comparison of tensors at different points of the manifold. In reality, co-