A GENERALIZATION OF RECURRENTS

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1. *Introduction.* It is well known that if

$$\phi(x) = \sum_{r=0}^{\infty} \phi_r x^r, \qquad \psi(x) = \sum_{s=0}^{\infty} \psi_s x^s$$

are two singly infinite series, then the coefficients in the expansion of $\phi(x)/\psi(x)$, $\log \phi(x)$, $e^{\phi(x)}$ can all be expressed as determinants in the quantities ϕ_r, ψ_s. These expressions are called *recurrents* and have been used by several writers* to evaluate determinants involving the binomial coefficients, Bernoulli numbers, etc.

In the present paper, the analogous results are given for the quotient of two *doubly* infinite series, and the logarithm and exponential of a doubly infinite series. The extension to m-tuply infinite series is briefly sketched in §8.

It is believed the expressions obtained are new; there is no reference to any such work in the four volumes of Muir's *History*. We assume throughout that all the series involved are absolutely convergent, so that the derangements and multiplications employed are justified. As a matter of fact, we are dealing essentially with infinite sets of quantities A_{rs}, B_{rs}, C_{rs}, $\cdots$, $(r, s, = 0, 1, 2, \cdots)$; the "variables" which appear in the series are merely convenient carriers for their coefficients.

We shall use, wherever convenient, the convention employed by writers on relativity for summations, namely,

$$U_{rs} x^r y^s,$$

which is taken to mean

$$\sum_{r=0}^{\infty} \sum_{s=0}^{\infty} U_{rs} x^r y^s,$$

the summations being understood.

* Muir's *History*, vols. II, III, IV, Chapters on recurrents.