GENERALIZATION OF THE BELTRAMI EQUATIONS TO CURVED n-SPACE*

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Let S be a curved n-space in which the linear element is given by the equation

$$ds^2 = \sum E_{ij} dx_i dx_j, \qquad (i,j = 1,2, \cdots ,n). \tag{1}$$

Without loss of generality, we may suppose

$$E_{ij} = E_{ji}. \tag{2}$$

Also let $U^{(i)}$, $(i=1, 2, \cdots, n)$, be a set of n independent functions of $x_1, x_2, \cdots, x_n$.

We shall say that the $U^{(i)}$ are isothermal in S provided they satisfy the relation

$$\sum (dU^{(i)})^2 = \lambda \sum E_{ij} dx_i dx_j, \tag{3}$$

where λ is a function of the x_i only.

If in (3) we express the $dU^{(i)}$ in terms of the differentials of $x_1, x_2, \cdots, x_n$ it follows from the independence of these differentials that the coefficients of corresponding terms on the two sides of the equation are equal and we obtain the $n(n+1)/2$ equations

$$\sum_{k=1}^{n} U_{xi}^{(k)} U_{xj}^{(k)} = \lambda E_{ij}. \tag{4}$$

Let D be the discriminant of the quadratic differential form in (1) and suppose it to be written as a determinant

$$|E_{ij}|, \tag{5}$$

in which E_{ij} is the element in the ith row and jth column. If each element of (4) be multiplied by λ and if for λE_{ij} be substituted its equal given by the left side of (4), we

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