A POINT SET WHICH HAS NO TRUE QUASI-COMPONENTS, AND WHICH BECOMES CONNECTED UPON THE ADDITION OF A SINGLE POINT*

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If M is a set of points, and P is a point of M, then the *quasi-component* of M determined by P is the set of points common to all possible sets M_1, where $M=M_1+M_2$, M_1 contains P, and M_1 and M_2 are mutually separated sets.† If M is a connected set, then the quasi-component of M determined by P is M itself. If M is not connected, then it can be considered as the sum of its quasi-components. If a quasi-component consists of more than one point, it is called a *true quasi-component*. If all the quasi-components of M reduce to single points, that is, if M contains no true quasi-components, then M is totally disconnected (i. e., has no connected subset consisting of more than one point). However, the quasi-components of a totally disconnected set do not necessarily reduce to single points. Sierpinski has given‡ an example of a set of points N and a point P not in N, such that N has no true quasi-components, and such that the set $N+P$, although totally disconnected, contains a true quasi-component consisting of P and a certain point of N.

Knaster and Kuratowski have given§ an example of a totally disconnected set of points S, which becomes connected upon the addition of a certain point a; i. e., $S+a$

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† See F. Hausdorff, *Grundzüge der Mengenlehre*, Leipzig, 1914, p. 248. Two sets are called *mutually separated* if they have no point in common, and if neither contains a limit point of the other.

‡ *Sur les ensembles connexes et non connexes*, Fundamenta Mathematicae, vol. 2 (1921), pp. 81–95.

§ *Sur les ensembles connexes*, Fundamenta Mathematicae, vol. 2 (1921), pp. 206–255. See especially pp. 240 ff.