Leçons sur les Propriétés Extrémales et la Meilleure Approximation des Fonctions Analytiques d'une Variable Réelle. By Serge Bernstein. Paris, Gauthier-Villars, 1926. 10+207 pp.

This book, which is one of the Borel series of monographs, forms an exposition of the course of lectures given by Professor Bernstein at the Sorbonne in May, 1923. The results are for the most part due to the author himself, and the majority of them appear here in print for the first time. The work as a whole is a development of the methods of Tchebycheff in the study of the field indicated by the title. The great variety of interesting and important results that are here contained in some two hundred pages, furnishes the best possible support for the writer's contention that the ideas of Tchebycheff are of fundamental importance in this domain of the theory of analytic functions of a real variable.

The first chapter is concerned with extremal properties on a finite segment of polynomials and other functions depending on a given number of parameters, and develops the algebraic basis for the whole subject. It contains among other important theorems a classical result of the author concerning the best approximation to $|x|$ by polynomials of given degree. We also find here his fundamental theorem with regard to the relationship between the maximum modulus of a polynomial and that of its derivative.

In the second chapter a study is made of extremal properties on the entire axis of reals of algebraic functions and of certain types of integral functions. In connection with the application of these properties to polynomial approximation there is introduced the notion of a "function of comparison", $\phi(x)$, and the extremal properties of the expression

$$\epsilon_n = \left| \frac{f(x) - P_n(x)}{\phi(x)} \right|,$$

where f is a given function and P_n an arbitrary polynomial of degree n, are studied. It will be seen that this use of a function of comparison is analogous to the notion of relatively uniform convergence introduced by E. H. Moore. We find also in this chapter the interesting theorem that any continuous function of a real variable, $f(x)$, such that $\lim_{x\to+\infty} f(x) = A$, can be uniformly approached on the whole axis of reals by means of rational fractions possessing given poles $\alpha_n \pm \beta_n$, provided the series $\Sigma|\beta_n|/(\alpha_n^2+\beta_n^2)$ diverges. On the other hand, the uniform approach to $f(x)$ by rational fractions, possessing poles such that the above series is convergent, implies the fact that $f(x)$ is an analytic function having no singularities in the finite plane other than poles at $\alpha_n \pm \beta_n$. In the latter part of the chapter the fundamental results regarding the relationship between the maximum of the modulus of a polynomial and that of its derivative, referred to in connection with Chapter II, are generalized to the case of integral functions.

The third chapter deals with the question of the best approximation to analytic functions possessing given singularities. As that topic had previously been treated by de la Vallée Poussin in his *Leçons sur l'Approximation des Fonctions d'une Variable Réelle*, also of the Borel series, Pro-