

OSGOOD ON ANALYTIC FUNCTIONS OF SEVERAL VARIABLES

Lehrbuch der Funktionentheorie. By W. F. Osgood. Part I. Leipzig, B. G. Teubner, 1924. v+242 pp.

It may appear strange to give a review of a part of a volume, an "*Erste Lieferung.*" There is, however, good justification for this. The present 242 pages make a substantial unit by themselves. They comprise the general concepts and theorems which form the basis for a general theory of analytic functions of two or more variables. Subsequent chapters to be added later will be devoted to special branches from the main trunk, such as the theory of $2n$ -ply periodic functions of n variables, theta-functions, or automorphic functions in more than one variable; and so on. Since also some considerable time will elapse before the writing and publication of these chapters, it seems both suitable and desirable to review the present brochure.

The need of an up-to-date coordination and systematization of material relating to analytic functions of several variables is almost too apparent to enlarge on. While many of the results and methods for analytic functions of one variable can be carried over without difficulty to functions of many variables, the difficulties at other points are so great, the pitfalls so hidden, the progress so slow, as to make it almost legitimate to say that a profound gulf exists between the theory of analytic functions for a single variable and that for several variables.

The literature pertaining to analytic functions of several variables is rather scattered, and little in the nature of systematic exposition is to be found. Chapter XVII in Goursat's *Cours d'Analyse* and Chapter IX in the second volume of Picard's *Traité d'Analyse* are excellent so far as they go, but are confined to the elementary aspects of the subject. Then there are the Calcutta lectures of Forsyth upon *Functions of Two Complex Variables* (1914), but so different are they in character, scope, and content from the work under review that they increase the desirability and usefulness of the latter. Synoptic reports on analytic functions of many variables are to be found in fourteen pages of the section written by Osgood a quarter-century ago for the *Encyklopaedie der Mathematischen Wissenschaften*, in Hartogs' Bericht in the *Jahresbericht der deutschen Mathematiker-Vereinigung* (vol. 16 (1907)), and finally in Bieberbach's recent encyclopaedia article (1921), giving the *Neuere Untersuchungen über Funktionen von komplexen Variablen*, of which, however, only a tenth is devoted to analytic functions of several variables. Lastly, I may cite as a forerunner of the present work the Madison Colloquium Lectures of Osgood (1914), treating *Topics in the Theory of Functions of Several Complex Variables*. Naturally, these lectures have also largely the nature of encyclopaedic sketches or skeleton outlines without the connective tissues.

Such a meager amount of literature reveals how strangely little the analytic function-theory has been in fashion mathematically, although its