EXTENSIONS OF WARING'S THEOREM ON FOURTH POWERS*

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1. *Introduction.* In 1770 Waring conjectured that every positive integer p is a sum of nineteen integral biquadrates. It is shown in § 8 that eight of them may be taken equal if $p \leqq 4100$. Again, sixteen of them may be taken equal in pairs if $p \leqq 2400$. All possible similar results are included in Theorem 1 of § 3.

2. *Notations and Definitions.* The form

$$(1) \qquad (a_1, \cdots, a_n) = a_1x_1^4 + \cdots + a_nx_n^4, \qquad (0 < a_1 \leqq a_2 \leqq a_3 \cdots),$$

is said to be of *order* n and *weight* $a_1+a_2+\cdots+a_n$. Since $ax^4 = x^4 + \cdots + x^4$, to a terms, a form of weight w is equal to a sum of w biquadrates. But 79 is not a sum of fewer than nineteen biquadrates. Hence 19 is the minimum weight of a form (1) which represents all positive integers.

Let f be a form (1) which represents p, and let $a_1 = r+s$. The form $g = (r, s, a_2, \cdots, a_n)$ shall be said to be derived from f by the *partition* of a_1 into $r+s$. If we give to the first two variables in g the same value x_1 as was employed in $f = p$, we see that also g represents p. Hence *any form derived from f by partition represents every integer which can be represented by f* (and usually represents further integers).

Write $a = 2^4$, $b = 3^4$, $c = 4^4$, $\cdots$. If a positive integer m can be expressed as a linear combination of 1, a, b, $\cdots$, with integral coefficients $\geqq 0$ whose sum is $\leqq 19$, in one and only one way, m shall be called a *simple* number. In case there are exactly two such expressions, m shall be called a *double* number. Similarly for a *triple* or *k-fold* number.

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