Again, generally $f_1 \neq f'$. Under what condition will $f_1 = f'$? From (2) and (3) we see that *the dual of a function is the same as the negative of the function if each discriminant is the dual-negative of its conjugate.*

Finally, if we have a relation $f = 0$, then in general $f_1 \neq 1$, though always $f' = 1$. What is the condition that $f_1 = 1$ when $f = 0$? By means of (2) and (3) we find this condition to be *the same as the condition that* $f_1 = f'$.

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THE HEAVISIDE OPERATIONAL CALCULUS*

BY H. W. MARCH

In a number of recent papers, Carson† has made a definite advance in the study of the Heaviside operational calculus by showing that the solution of an operational equation of the type in question can be obtained from an integral equation. Having done this, he was able to discuss Heaviside's three principal rules and to derive a number of important theorems by the use of which it is possible to solve by operational methods, problems to which Heaviside's rules are not directly applicable.

Somewhat earlier Bromwich‡ and Wagner§ solved, by the use of contour integrals in the complex plane, problems to which one of Heaviside's rules is applicable. They noted that the corresponding rule of Heaviside, the expansion theorem, follows at once from a calculation of the residues at the poles of the integrand in the case of a suitably restricted

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† J. R. Carson, Bell Technical Journal, vol. 4 (1925), pp. 685–761; vol. 5 (1926), pp. 50–95, 336–384; this Bulletin, vol. 22 (1926), pp. 43–68. See also P. Levy, Bulletin des Sciences Mathématiques, vol. 50 (1926), pp. 174–192.

‡ T. J. I'A. Bromwich, Proceedings of the London Society, (2), vol. 15 (1916), pp. 401–448; see also H. S. Carslaw, Philosophical Magazine, vol. 39 (1920), pp. 603–611.

§ K. W. Wagner, Archiv für Elektrotechnik, vol. 4 (1916), pp. 159–193.