THE DUAL OF A LOGICAL EXPRESSION*

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The Peirce-Schröder law of duality in Boolean logic is that for every proposition in the logic there exists another proposition, obtained from the former by interchanging the operations $+$, $\times$ and the elements 0, 1. The rule for obtaining the dual of a logical expression involved in this law is convenient enough when no special form is desired for the dual, but it is generally not at all convenient if the dual is required in the very much desired *normal* form. The main object of this note is to obtain a convenient rule for writing down the dual of an expression in the normal form.

Let $a, b, \cdots, l$ be the *discriminants* of a logical function $f(x, y, \cdots, t)$. Then f, developed normally with respect to its arguments, is given by

$$(1)\quad f(x,y,\cdots,t) = axy\cdots t + bxy\cdots t' + \cdots + l x'y'\cdots t',$$

where the primes indicate negation. If f_1 denote the dual of f, we have by the rule of Peirce and Schröder

$$f_1 = (a_1 + x + y + \cdots + t)(b_1 + x + y + \cdots + t')\cdots (l_1 + x' + y' + \cdots + t'),$$

where $a_1, b_1, \cdots, l_1$ are the respective duals of $a, b, \cdots, l$.† Or, developed normally with respect to $x, y, \cdots, t$,

$$(2)\qquad f_1 = l_1xy\cdots t + \cdots + b_1x'y'\cdots t + a_1x'y'\cdots t'.$$

Let us call two discriminants of a function *conjugate* when the arguments associated with one are the respective negatives of those associated with the other. Then (2) tells us

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† The discriminants $a, b, \cdots, l$ may be 0, 1, or functions of other elements, $\alpha, \beta, \cdots, \mu$; so that $a_1, b_1, \cdots, l_1$ will in general be different from $a, b, \cdots, l$ respectively.