

## INVARIANTS OF A PORISTIC SYSTEM OF TRIANGLES\*

BY J. H. WEAVER

1. *Introduction.* In the last quarter of a century considerable work has been done on various systems of triangles.† Several fixed circles have been discovered which are associated with poristic systems of triangles. However, so far as the writer knows, no attempt has been made to determine all the invariant curves of any degree for any system of triangles. The present paper determines the invariant curves of the first and second degrees for a poristic system of triangles which has a fixed incircle and a fixed circumcircle.

2. *General Considerations.* Let there be a triangle  $ABC$  with a fixed incircle, center  $I$ , and a fixed circumcircle, center  $O$ . If we consider  $OI$  as the  $X$ -axis and  $I$  as the origin, the equations of the sides of the triangle may be written in the normal form as follows:

$$(1) \quad \begin{cases} BC: & x \cos A_1 + y \sin A_1 - r = 0, \\ CA: & x \cos A_2 + y \sin A_2 - r = 0, \\ AB: & x \cos A_3 + y \sin A_3 - r = 0, \end{cases}$$

where  $r$  denotes the radius of the incircle.

If  $A'B'C'$  be any other triangle of the system the equations of its sides are

$$(2) \quad \begin{cases} B'C': & x \cos A'_1 + y \sin A'_1 - r = 0, \\ C'A': & x \cos A'_2 + y \sin A'_2 - r = 0, \\ A'B': & x \cos A'_3 + y \sin A'_3 - r = 0. \end{cases}$$

Let  $P(x', y')$  be any point in the plane of the triangles. Then the distances of  $P$  from the sides (1) and (2) are

$$(3) \quad \alpha_i = x' \cos A_i + y' \sin A_i - r, \quad (i = 1, 2, 3),$$

$$(4) \quad \alpha'_i = x' \cos A'_i + y' \sin A'_i - r, \quad (i = 1, 2, 3),$$

respectively.

---

\* Presented to the Society, September 8, 1926.

† See Gallatley, *Modern Geometry of the Triangle*, second edition, London, 1910; and Coolidge, *Treatise on the Circle and Sphere*, Oxford, 1916.