FUNCTIONS EXPANSIBLE IN SERIES*

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In the Transactions of this Society† Hopkins has stated and proved the following theorem.

THEOREM I. *If $f(x)$ is a function analytic in the interior and on the boundary of a circle centered at $x=0$ and of radius x_0, $0<x_0<\pi$, which involves in its power series expansion only powers of x of indices congruent to* 2 (mod 3), *and which has a continuous second derivative for real values of x in the interval $0 \leqq x \leqq \pi$, then the formal development of $f(x)$ in a series whose terms are the characteristic functions of the differential system*

$$\frac{d^3u}{dx^3} + \rho^3 u = 0, \qquad u(0) = u'(0) = u(\pi) = 0,$$

converges uniformly to $f(x)$ in the interval $0 \leqq x \leqq x_0$.

Hopkins proved further that the development converges uniformly to $f(x)$ in the interior of an equilateral triangle centered at $x=0$ and having one vertex at $x=x_0$. The following theorem in which the adjoint differential system appears is obtained from Theorem I by the change of variable $x'=\pi-x$.

THEOREM II. *If $f(x)$ is a function analytic in the interior and on the boundary of a circle centered at $x=\pi$ and of radius x_1, $0<x_1<\pi$, which involves in its power series expansion only powers of $(\pi-x)$ of indices congruent to* 2 (mod 3), *and which has a continuous second derivative for real values of x in the interval $0 \leqq x \leqq \pi$, then the formal development of $f(x)$ in a series*

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* Presented to the Society, April 2, 1926.

† J. W. Hopkins, Transactions of this Society, 1919, pp. 245, et seq. Published by D. Jackson.