A METHOD FOR ACCELERATING THE CONVERGENCE IN THE PROCESS OF ITERATION*

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1. *Introduction.* The simplicity and directness of the process of iteration coupled with the fact that errors committed along the way do not vitiate the final result have won for the method a certain degree of widespread popularity. This has become imperiled, however, by the extreme slowness of the convergence in many cases. The object of the present paper is to overcome the difficulty by furnishing a powerful method based on the same kind of analysis as Newton's but better adapted to the process of iteration. The common practice of taking half the sum of two successive approximations, one of which is too large and the other too small, is seen to be inadequate by the following example: $x=2+\pi \sin x$. Here one iterates using the formula $\sin x_2=(x_1-2)/\pi$. For x_1 close to the true root $m_1\equiv dx_2/dx_1=1/(\pi \cos x_2)=-.331$ nearly. The half sum is no improvement here and one finds the same value for the ninth application of the formula as by unmodified iteration, provided one starts with x_1 as the equivalent of 164° in radians. This value is obtained by two applications of the new method. Its equivalent in degrees is 164.05131, and is correct to five decimals.

By ordinary iteration it requires about a score of approximations to solve $6k+10e^{-k}=10$† correct to six decimals. The result, $k=1.126261$, is found by two applications of the new method by starting with $k_1=1.1$, obtained graphically. Here $m_1=.548$. One can use the half sum only when m is negative; and if m is near $-1/3$, no improvement results. The present method consists of one iteration and one application of formula (8), which may be regarded as a formula of interpolation

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† See Phillips, *Differential Equations*, p. 15, ex. 5.