THE ASYMPTOTIC OSCULATING QUADRICS OF A CURVE ON A SURFACE

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1. *Definition.* Let us consider a surface S, a curve C on S, and three neighboring points P, P_1, P_2, on C. The three tangents at these points to the asymptotic curves of one family determine a quadric whose limit, as P_1, P_2 independently approach P along C, is a quadric called* by Bompiani an *asymptotic osculating quadric* of C at P. A second asymptotic osculating quadric is obtained by using the other family of asymptotics. We shall now derive the equations of these quadrics, using Wilczynski's notation, and shall deduce some of their fundamental properties.

2. *Equations.* Let the four homogeneous coordinates y of a point P on a non-degenerate non-developable surface S be given as analytic functions of two independent variables u, v; and let the curves $u=\text{const.}$, $v=\text{const.}$ be the asymptotics. Then the functions y, when multiplied by a suitably chosen proportionality factor, are solutions of Wilczynski's canonical system of differential equations,

$$y_{uu} + 2by_v + fy = 0, \quad y_{vv} + 2a'y_u + gy = 0. \tag{1}$$

The one-parameter family of curves on S represented by the equation

$$dv - \lambda du = 0 \tag{2}$$

contains one curve C through P. The coordinates Y of any point P_1 on C near P are given by an expansion of the form

$$Y = y + \frac{dy}{du}\Delta u + \frac{1}{2}\,\frac{d^2y}{du^2}\Delta u^2 + \cdots .$$

* Bompiani, *Geometria delle superficie considerate nello spazio rigato*, Rendiconti dei Lincei, 1926.