La Géométrie des Espaces de Riemann. (MÉMORIAL DES SCIENCES MATHÉMATIQUES, No. 9.) By Élie Cartan. Paris, Gauthier-Villars, 1925. 59 pp.

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The aim is to present in brief form the developments of Riemannian geometry and especially that which came as a consequence of Levi-Civita's notion of parallelism. Using this, the author is able to present the subject with a minimum of formal work and a maximum of geometry. It is a delight to a geometer to read this geometric argument instead of solid pages of formulas which confront one so often. The first part is in sufficient detail that one follows the argument easily and after that many of the theorems are not proved but the statement is so clear and concise that one does not feel the need of proof. The notion of covariant and contravariant systems is assumed. Chapter I presents euclidean geometry in curvilinear coordinates and discusses especially the change in coordinates of a point when the reference system with origin at M is moved into a reference system with origin at a nearby point M'. These changes are given by the formula

$$dx^i + du^i + \Gamma^i_{kr}\, X^k\, du^r = 0$$

where du^i are the coordinates of M' with respect to the coordinate system with origin at M. The left side of the formula can also be interpreted as the change of the coordinates of a moveable point or a vector. Thus

$$D_r x^i = \frac{\partial x^i}{\partial u^r} + \overset{i}{\epsilon}_r + \Gamma^i_{kr}\, x^k, \quad \overset{i}{\epsilon}_r = 1 \text{ if } i = r, \quad \overset{i}{\epsilon}_r = 0, \text{ if } i \neq r$$

$$D_r X^i = \frac{\partial X^i}{\partial u^r} + \Gamma^i_{kr}\, X^k$$

where x^i are the coordinates of a point and X^i of a vector. The Γ's are then shown to be Christoffel symbols of the second kind.

Chapter II then discusses Riemannian space and shows that an osculating euclidean space exists at each point, which will contain two near by reference systems and hence the above formulas can be applied to this space. This leads to the Levi-Civita parallelism and the rest of the booklet is an application of these notions.

To give a more definite notion of the subjects treated I give the titles of the chapters. Chapter I, curvilinear coordinates in euclidean geometry; Chapter II, Riemannian spaces; Chapter III, Riemann-Christoffel tensor and Riemannian curvature; Chapter IV, Bianchi's identities, vector curvature, spaces of constant curvature; Chapter V, totally geodesic varieties; Chapter VI, varieties immersed in a Riemannian space; Chapter VII, class, degrees of freedom, holonomic groups.

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