WHITEHEAD AND RUSSELL'S PRINCIPIA MATHEMATICA

Principia Mathematica. By Alfred North Whitehead and Bertrand Russell, Volume I. Second Edition. Cambridge, University Press, 1925. xlvi+674 pp.

The second edition of Volume I of Whitehead and Russell's *Principia Mathematica* leaves the first edition intact (except for minor changes), but adds some new sections. The new sections—an introduction and three appendices—are chiefly devoted to a restatement of the logical theories of the authors in the light of the reduction in the number of primitives in the *Principia* by Sheffer and Nicod and in the light of the views of Wittgenstein (expressed in his remarkable *Tractatus Logico-Philosophicus*) on propositions and propositional functions.

The authors announce in the new Introduction the main improvements which they find necessary to make in their logic. These improvements I may list as follows: the dropping of the distinction between "real" and "apparent" variables; the dropping of the primitive idea "assertion of a propositional function"; the reading, on all occasions, of "$\vdash \cdot fp$" as "$\vdash \cdot (p) \cdot fp$"; the dropping of the primitive proposition *1·11. The authors, also, apparently abandon their "axiom of reducibility," because "clearly it is not the sort of axiom with which we can rest content," though, because of this abandonment, "there is, so far as we can discover, no way by which our present primitive propositions can be made adequate to Dedekindian and well-ordered relations." I have not included in the above list of changes the Sheffer-Nicod simplification of the primitives of the *Principia*. This simplification, elegant as it is from a certain point of view, is not a logical necessity. The general logical make-up of the old *Principia* is not affected by the revised edition. The authors begin with a set of primitives in the logic of "elementary" propositions, whose independence (and consistency) are left unproved because "the ordinary methods of proving independence are not applicable, without reserve, to fundamentals," and from these primitives, together with primitives introduced later, they aim to derive, in a definitely restricted way, the rest of logic and all mathematics.

When one considers the caliber of our authors and the fact that the *Principia* has occupied a prominent place on mathematical shelves for fourteen years, one wonders that the book has influenced mathematics so little. Of course, a partial explanation lies in the magnitude and structure of the *Principia*. Volume I, which is only one of four royal octavo volumes, which deals only with mathematical logic and matters introductory to the theory of cardinals, contains over 600 pages (over 700 in the new edition), has over 400 different symbols (some of them representing extremely subtile ideas), and has thousands of propositions, wholly written and demonstrated in symbols and linked together in an unbreakable chain. But the chief reason for the aloof attitude of mathematicians toward