A CURIOUS IRREDUCIBLE CONTINUUM

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One of Janiszewski's theorems* is to the effect that, if ab is a bounded irreducible continuum and c is a point of the first species, there is a decomposition of ab into two continua, ac and cb, such that $ac \cdot cb = c$. The fact that the condition imposed on c is not a necessary one naturally led to the question as to whether an irreducible continuum not made up of indecomposable continua can always be decomposed into two continua having only one point in common. A vain endeavor to answer this question in the affirmative resulted in the following example to the contrary, which may be of interest to workers in this field.

Although the continuum is simple, it is difficult to describe without a figure and therefore the construction should be carried out through the first two stages.

The first stage in the construction is as follows. Take a unit square in the first quadrant and mark the following points: $a=(0, 0)$, $b=(1/4, 0)$, $c=(1/2, 0)$, $d=(3/4, 0)$, $e=(1, 0)$, $f=(1/2, 1/4)$, $g=(3/4, 1/4)$, $h=(1/4, 3/4)$, $i=(1/2, 3/4)$, $j=(0, 1)$, $k=(1/4, 1)$, $l=(1/2, 1)$, $m=(3/4, 1)$, $n=(1, 1)$. Draw the straight lines bk, cf, fh, gi, il, and dm. Set $E_1=bk+cf+fh$ and $E_2=gi+il+dm$. The continuum E_1 may be regarded as the union of two continua, bk and $cf+fh+hk$, extending from ae to jn and having a common segment hk of length 1/4. A similar statement holds for E_2. The continua E_1 and E_2 divide the square into five "strips," of which two, $bcfh$ and $ligm$, meet only one side of the square and are called *incomplete*. The other three extend from ae to jn and are called *complete*; two of them are rectangles, $R_1=abkj$ and $R_2=denm$, and the third is a polygon $P_1=$

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* Z. Janiszewski, *Sur les continus irréductibles entre deux points*, JOURNAL DE L'ÉCOLE POLYTECHNIQUE, (2), vol. 16 (1926), p. 125.