THE TRANSVERSALITY RELATIVE TO A SURFACE OF $\int F(x, y, z, y', z')dx = $ MINIMUM*

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1. *Introduction and Statement of Theorem.* Consider, for illustration, a surface Σ, on which A and B are any two points. Then, of all curves in space which join A and B, the straight line segment AB is the shortest; of all curves on Σ joining A and B, the geodesic AB is the shortest.

The generalization from the length integral

$$s = \int (1 + y'^2 + z'^2)^{1/2} dx \tag{1}$$

to the general integral of first order

$$J = \int F(x, y, z, y', z') dx \tag{2}$$

is obvious. Thus we may speak on the one hand of the unrestricted extremals of J relative to space, ∞^4 in number, and, on the other, of the extremals of J relative to a given surface Σ, ∞^2 in number.

The idea of *transversality* may likewise be defined relative to a given surface as well as for space. Let us review the well-known definitions and facts in this connection. The space transversality T belonging to an integral J is essentially a correspondence between lineal elements and surface elements (of the first order) characterized by the following two properties: (1) a lineal element and its corresponding surface element have the same base point, (2) if, taking an arbitrary base surface S, we construct the ∞^2 extremals of J which meet S transversally, then lay off along each extremal, starting at S, an arc over which the integral J has a fixed value, the locus of the end points of these arcs is a surface trans-

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