ON SOME THEOREMS OF BÔCHER CONCERNING ISOLATED SINGULAR POINTS OF HARMONIC FUNCTIONS*

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In a note entitled *Deux théorèmes élémentaires sur les singularités des fonctions harmoniques,*† Picard gave proofs of the following theorems.

I. *If $u(x, y)$ is single-valued and harmonic in the neighborhood of a point A, except at A, and is bounded, then $u(x, y)$ becomes harmonic at A also when its definition at this point is properly adjusted.*

II. *If $u(x, y)$ is single-valued and harmonic in the neighborhood of A, except at A, and becomes positively infinite at A, then $u(x, y) - h \log (1/r)$ is harmonic at A, where h is a properly chosen constant, and r is the distance of (x, y) from A.*

As credit for these results was not assigned in the note cited, the author must have been unaware of a paper by Bôcher,‡ in which were proved the above theorems, and several others, not only for the plane, but for n dimensions, and for other differential equations of the elliptic type as well as Laplace's.

In addition to calling attention to the above matter of authorship, the object of this paper is to give an elementary proof of a theorem from which follow at once the results given by Bôcher, as far as solutions of Laplace's equation

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