# ALL INTEGRAL SOLUTIONS OF $ax^2+bxy+cy^2=w_1 w_2 \cdots w_n$*

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1. *Literature.* This Diophantine equation (or cases of it) has been treated in two papers by the writer and two by Professor Wahlin, all published in this BULLETIN†. Three of these papers were based on the theory of algebraic ideals. The writer's paper of 1923 employed an elementary method to find all integral solutions of $x^2-my^2=zw$. The present paper is elementary and is a sequel to the latter paper.

2. *Reduction to the Case $n=2$.* Let $q$ denote a quadratic form in $x$ and $y$. The problem to solve $q=zw$ shall be called the homogeneous problem. To it will be reduced the problem to solve $q=w_1 \cdots w_n$. Write $z=w_1 \cdots w_{n-1}$. By our solution below of the homogeneous problem $q=zw_n$, $x, y, z, w_n$ are products of an arbitrary integer $h$ by certain functions $X, Y, Z, W$ of certain parameters, only two of which, say $\xi$ and $\eta$, occur in the quadratic expression $Q(\xi, \eta)$ for $Z$. Since $w_1 \cdots w_{n-1}=hZ$, evidently $w_i=h_iW_i$ $(i=1, \cdots, n-1)$, where the $h_i$ are integers whose product is $h$. Hence $q=w_1 \cdots w_n$ is reduced to the solution of $Q(\xi, \eta)=W_1 \cdots W_{n-1}$, which is of the form of our initial equation with $n$ replaced by $n-1$. The resulting values of $\xi$ and $\eta$ in terms of new parameters are to be inserted in the functions $X$, $Y$, and $W$.

3. *Simplification of the Homogeneous Problem.* The greatest common divisor $\omega$ of the coefficients of $q(x, y)$ must

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