A CHARACTERISTIC PROPERTY OF MINIMAL SURFACES*

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1. *Statement of Theorems.* By the mean curvature of a surface at a point we shall understand $1/R_1+1/R_2$, where $1/R_1$, $1/R_2$ are the principal curvatures at the point. There is some divergence in the literature with respect to this nomenclature; some authors call $(1/R_1+1/R_2)/2$ the mean curvature.‡

Imagine that about any point O of an arbitrary surface Σ as center we describe a sphere S of infinitesimal radius r. Then S is intersected by Σ in a curve C infinitely close§ to the great circle cut from S by the tangent plane to Σ at O. The curve C divides the surface of S into two areas nearly equal to each other. Also, the portion of Σ lying within S divides S into two nearly equal volumes.

The present paper is devoted to the proofs of the following three theorems.

THEOREM I. *Let the difference between the two areas into which C divides S be denoted by η. Then, in the limit as $r\rightarrow 0$,*

$$\frac{\eta}{V}=\frac{3}{4}\left(\frac{1}{R_1}+\frac{1}{R_2}\right)$$

where V denotes the volume of S.

THEOREM II. *Let v denote the difference of the two volumes into which S is divided by Σ. Then, in the limit as $r\rightarrow 0$,*

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‡ Bianchi, Eisenhart, and Scheffers use the first notation; Blaschke and Darboux the second.

§ Even in comparison with the radius of the sphere, that is, the deviation of C from the great circle is an infinitesimal of the second or higher order.