

## WHITE ON CUBIC CURVES

*Plane Curves of the Third Order.* By H. S. White. Published with the cooperation of the National Research Council. Cambridge, Harvard University Press, 1925. xii+168 pp. \$2.75.

The author's purpose in writing this book clearly has been to produce, not a detailed treatise on cubic curves, but a clear, readable, and rigorous textbook explaining the outstanding projective properties of these curves. Thus his fundamental problem has been throughout, to select judiciously from a wealth of available material, and to present skillfully, the main features of the theory of cubic curves. In this he has succeeded admirably. The style is nowhere hurried, and the proofs are painstakingly thorough; yet the essentials of the subject are presented in a volume that is by no means lengthy. The author hopes that this work "may be found helpful as a stepping-stone to many extensive and beautiful treatises on special themes." For this purpose a considerable number of references are given.

The book starts with a study of the properties of real cubic curves invariant under real projective transformations. This is followed by a discussion of the simplest theorems on the intersections of cubics with other curves, especially with straight lines. There follows a discussion of considerable length on the properties of the chief invariant loci. Symbolic notation is here introduced and used, but the elaborate formal machinery of this notation is, as far as possible, deliberately and skillfully avoided. Several methods of generating the general cubic are then given, after which the problem of geometry on a cubic is approached, in the most natural and logical manner, by establishing and using Noether's fundamental theorem. This is followed by the parametric representation of the non-singular cubic by means of elliptic functions, with various applications showing the power of this representation. The final chapters bring in the notion of apolarity and point out some of the consequences of this theory.

One who wishes to secure as easily as possible an insight into the spirit and the methods of the projective theory of higher plane curves will find that a preliminary study of cubic curves constitutes an excellent introduction to the field. Most of the principal theories and modes of reasoning are duly exemplified, but in a preliminary and simplified form. The irrationality of the non-singular cubic, for example, introduces difficulties that can be mastered *simply* and rigorously by the use of Noether's  $Af+B\phi$  theorem for cubics and the parametric representation of the curve by means of elliptic functions,—concepts both of which the student will find to be of fundamental importance in more advanced work. For an introduction of this character, the text under consideration is conspicuously well adapted. The presentation of the various topics makes full use of the peculiarities of cubic curves, yet the reasoning involved is of the type that the reader will need in the more difficult study of curves of higher order