PRINGSHEIM ON COMPLEX VARIABLES

Vorlesungen ueber Zahlen- und Funktionenlehre. Vol. II, pt. 1: *Grundlagen der Theorie der analytischen Funktionen einer komplexen Veraenderlichen.* By Alfred Pringsheim. Leipzig, B. G. Teubner, 1925. xv+626 pp.*

In laying the foundations for analytic functions of a complex variable it is possible to proceed from three practically equivalent points of view, the Meray-Weierstrass, which bases its developments upon the properties of power series, the Cauchy-Riemann point of view, in which the functions to be considered are limited by the existence of a differential quotient, and what might be called the Cauchy-Morera point of view, in which the properties of the function depend upon the properties of the curvilinear integral. Of these, the first two seem to be the only ones which are practicable, and are usually considered simultaneously, each contributing towards the development of a simple and elegant theory. Only occasionally do we find the one emphasized to the almost complete exclusion of the others.

The volume under discussion has for its avowed purpose the development of the foundations of the theory of analytic functions as far as possible along the Meray-Weierstrass lines, i.e., it is based on power series and their properties. This is probably a consistent point of view, in consideration of the fact that this volume is the second in a series, the first of which was devoted to series and sequences of numbers. The choice and arrangement of material is governed then by the desire to accomplish as much as possible by the use of power series.

A brief survey of the contents of the book may be of value.

The first chapter, of an introductory character, is devoted to the real variable and some of the essential properties of functions of real variables. We find, then, discussions of the one to one correspondence between real numbers and points on a line, limits of functions, and properties of continuous functions. The introduction of two variables leads to a consideration of regions and their boundaries, and this in turn to the Jordan Curve Theorem. The method of attack is via the properties of what Pringsheim calls "Treppenpolygone," which one might translate "stair polygons," consisting of segments of straight lines parallel to the coordinate axes, no point on the boundary occurring twice. This section is of a rather more advanced character than most of the remainder of the book. There is further a brief treatment of double and iterated limits and corresponding properties of functions. The chapter closes by indicating as the reason for adopting the power series as the form of function to be considered, its property of being determined throughout its region of existence by its values at an infinity of arbitrarily chosen points, and the desirability of using complex numbers, as giving perhaps a method for continuing power

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