NOTE ON HOROSPHERES*

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In a paper† before the Mathematical Association of America (Sept. 1923) and in another before the International Mathematical Congress at Toronto (August, 1924) I have developed a treatment of a part of non-euclidean geometry which has certain advantages, so it seems to me, over the projective methods of Klein. This method rests on the metric of Riemann and on the introduction of certain variables in terms of which the equations of the straight line and plane are linear.

In the first paper mentioned I showed how easy it is to arrive at Clifford's parallels in elliptic space. These lie on surfaces called Clifford surfaces; their curvature is zero and hence their geometry for restricted regions is euclidean.

In hyperbolic space there are also surfaces of zero curvature, the horospheres of Lobatschevsky and Bolyai. I now wish to show how they may be obtained by the preceding method.

Let x, y, z be ordinary rectangular coördinates. Let R be a positive constant. We set

$$r^2 = x^2 + y^2 + z^2,\ \lambda = 4R^2 - r^2,\ \mu = 4R^2 + r^2,$$
$$d\sigma^2 = dx^2 + dy^2 + dz^2. \tag{1}$$

The metric of H-space‡ as defined by Riemann is given by the equation

$$ds = \frac{4R^2 d\sigma}{\lambda}\cdot \tag{2}$$

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† American Mathematical Monthly, vol. 30, p. 425, and vol. 31, p. 26. The Proceedings of the Toronto Congress have not yet appeared.

‡ For H-, read hyperbolic; for e-, read euclidean.