NOTE ON THE MERSENNE NUMBER $2^{139}-1$*

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Lucas' test for the primality of the Mersenne number 2^n-1, n a prime number, has been applied to the number $2^{139}-1$. For $n=139$ we form the residues, modulo $2^{139}-1$, of the successive terms of the recurring series 3, 7, 47, 2207, $\cdots$, in which each terms is 2 less than the square of the preceding term. Then $2^{139}-1$ is composite if no one of the first 138 terms of this series is divisible by $2^{139}-1$. Since this proved true, this 42-digit number is composite; no factor of it is known although the search for such has been pushed to the limit 1,000,000 by Cunningham and Gérardin.†

Contrary to the suggestions of Lucas and other workers in this field, the test was made to the base ten instead of to the base two. Lucas' method,‡ in which the main operation is copying, would require the setting down of more than 2,500,000 digits. Mason§ has suggested an abbreviation of this method in which the copying is replaced by the quicker but more dangerous procedure of counting. Both these methods are too slow for dealing with numbers as large as $2^{139}-1$. The method employed in the present case was a variation of the writer's "cross-division"|| process adapted to meet the requirements of the calculation. The work was done entirely by a computing machine in 60 hours. Since Lucas' test for a composite Mersenne number is a negative one it is vital that no error be made. To minimize the chance of error, two checks were made at each step by casting out multiples of 1001.

It should be noted that the composite character of $2^{139}-1$ is in accord with the famous statement of Mersenne in 1644.

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* Presented to the Society, San Francisco Section, June 12, 1926.

† BRITISH ASSOCIATION REPORTS, 1911, p. 321.

‡ *Récréations Mathématiques*, vol. 2, 1883, p. 230.

§ PROCEEDINGS INDIANA ACADEMY OF SCIENCE, 1914, pp. 429–31.

|| AMERICAN MATHEMATICAL MONTHLY, 1926, p. 198.