SOME RECENT WORK IN THE CALCULUS OF VARIATIONS*

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I. THE SIMPLEST PROBLEM.

1. *First Order Conditions.* The simplest problem of the calculus of variations is concerned with an integral of the form

$$\int_{x_1}^{x_2} f(x, y, y')dx, \tag{1}$$

in which f is a function of class C'''† in a region R_1 of 3-space, determined by the conditions that (x, y) be in some region R of the xy-plane and y' be finite. The problem may then be formulated as follows.

To determine among all functions $y=y(x)$, $x_1 \leqq x \leqq x_2$, which (1) are of class C'; (2) satisfy the conditions $y(x_i)=y_i$, $i=1, 2$; and (3) are such that the points $(x, y(x))$ for $x_1 \leqq x \leqq x_2$ lie in the region R, a function $y_0(x)$ for which there exists a positive number d, such that all functions $Y(x)$, $x_1 \leqq x \leqq x_2$, which satisfy conditions (1), (2), and (3), and the further condition that $|Y(x)-y_0(x)| \leqq d$ for $x_1 \leqq x \leqq x_2$, give to the integral (1) a value not less (not greater) than the value which this integral has for $y_0(x)$. Here it is understood that y' denotes the derivative dy/dx. A function which satisfies the conditions (1), (2), and (3) is called an *admissible function.*

The classical procedure in solving this problem is as follows. We suppose that we have found a solution $y=y_0(x)$,

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† A function is said to be of class $C^{(n)}$ if it possesses continuous derivatives of orders $1, 2, \cdots, n$; a function is said to be of class $D^{(n)}$, if it is continuous, and consists of a finite number of parts of class $C^{(n)}$.