THE BOREL THEOREM AND ITS GENERALIZATIONS*

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That any one should attempt to devote a paper to the subject of the Borel Theorem may at first glance seem a presumption. A brief investigation will however reveal the following facts. (a) The Borel Theorem is closely related to the fundamental postulates of linear order. (b) There are many extensions and analogs of the Borel Theorem, some of which are hidden away in papers on other subjects. (c) The Borel Theorem has held and still holds a central position in the development and analysis of general spaces.

The arrangement of topics in the paper is suggested by the previous paragraph. No claim is made for completeness with respect to the extensions and analogs of the Borel Theorem, due to the nature of the case. Nor do I claim any originality in the material presented. I hope that a systematic treatment of the Borel Theorem in general spaces will be suggestive and perhaps create further desirable interest and results in these spaces.

I. THE BOREL THEOREM AND ITS EXTENSIONS FOR THE LINEAR INTERVAL AND n-DIMENSIONAL SPACE

In order to give a simple statement of the Borel Theorem I shall use the phrase "a family $\mathfrak{F}$ of intervals I covers the point-set E" to mean that every x of E is interior to some interval I_x of $\mathfrak{F}$. Then the Borel Theorem in its simplest form may be stated as follows.

If the family $\mathfrak{F}$ of intervals I covers the closed interval (a, b) then a finite subfamily of $\mathfrak{F}$ covers (a, b).

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