

it avoids the use of the transformation of Weierstrass which is exceedingly ingenious and interesting, but complicated and not easy to generalize for spaces of higher dimensions. The method used by Vivanti is the historically very important one which has come to us through a process of development from the ideas in Legendre's fundamental memoir on the second variation.

It seems to me that the theoretical material in this interesting book on the calculus of variations is presented in a somewhat less clear and systematic style than that which one finds in the other very important and useful treatises which we have had from the pen of Professor Vivanti. This is doubtless due to the author's justifiable desire to arrive as rapidly as possible at the results which are of significance for the interesting special problems which he considers. His conciseness has led to some inaccuracies, as in the reference on page 132 to Bolza for a part of the proof of the Euler-Lagrange multiplier rule. The text seems to imply a theorem which Bolza has not proved and which I do not believe to be correct. Limitations of space probably forbade the expansion of the material in the appendix. It is at present a very interesting outline of a chapter in the calculus of variations which is still in process of development. Professor Vivanti there regards the integrals of the calculus of variations as special instances of functions of lines, and applies to them Volterra's definition of the derivative of such a function. In order to be effective for these integrals, however, the definition must be modified slightly. To secure the derivative limit, one must make the first and second derivatives of the variation  $\eta(x)$  approach zero, as well as impose the Volterra requirement that the variation itself and the interval on which the variation is different from zero are infinitesimals.\* I believe that the differential corresponding to Volterra's derivative, or the differential of Fréchet, for a function of a line, would prove to be more convenient than the derivative in this connection. The definition of a generalized integral on page 289 needs modification if it is to be applicable to the non-parametric integrals which are considered in the preceding paragraphs.

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*Maschinenbauliche Beispiele für Konstruktionsübungen zur darstellenden Geometrie.* By Theodor Schmid. Leipzig and Vienna, Franz Deuticke, 1925. 52 pp., 25 plates.

This is a collection of a number of very simple examples in mechanical drawing for the use of the young "technician," who "as a rule is not able to apply the instruction which he got in the usual courses in descriptive geometry to practical examples."

To judge from the practical examples represented on the 25 plates, the courses in descriptive geometry which do not enable a student to apply the knowledge gained to the representation of such simple objects must indeed be in a bad state. In the opinion of the reviewer very little descriptive geometry is needed to understand such plates.

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\* Proceedings of the NATIONAL ACADEMY OF SCIENCE, vol. 1 (1915), p. 173.