REPRESENTATION OF INTEGERS BY CERTAIN TERNARY CUBIC FORMS*

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1. *Introduction.* Apart from Eisenstein's theory relating to his canonical form† comparatively few results are available on representation of integers by ternary cubic forms. By methods differing from those employed by Eisenstein, this paper develops theory relating to two such forms. One form and the associated theory is, however, obtainable from Eisenstein's results. The other form cannot be so obtained.

2. *Rational Prime Factors of Norms of Integers of Cubic Number Fields.* Let $K(x)$ be the algebraic number field defined by a root x of an irreducible cubic. Consider any integer π of $K(x)$. Its norm, $N(\pi)$, is a rational integer. We seek the properties of its rational prime factors.

Let $N(\pi)=p_1^i p_2^j \cdots$, where $p_1,p_2, \cdots$ are rational primes and $i,j, \cdots$ are positive integers. Let π_1 be a prime factor of π. Then π_1 divides $N(\pi)$ and hence divides one of its rational prime factors, say p_1. It could not divide two such factors, for then it would divide their greatest common divisor and hence would be a unit.

Let us set $p_1=\pi_1\alpha$, where α is an integer of $K(x)$. Then $N(\pi_1)\cdot N(\alpha)=N(p_1)=p_1^3$. Four apparent possibilities arise:

$$(1) \qquad N(\pi_1)=\pm 1, \qquad N(\alpha)=\pm p_1^3.$$

This must be excluded, as it would make π_1 a unit.

$$(2) \qquad N(\pi_1)=\pm p_1, \qquad N(\alpha)=\pm p_1^2.$$

Then $\pi_1\pi_1'\pi_1''=\pm p_1=\pm\pi_1\alpha$. It follows that $\alpha=\pm\pi_1'\pi_1''$, where π_1' and π_1'' are the conjugates of π_1. In this case p_1 is the norm of a prime of $K(x)$.

$$(3) \qquad N(\pi_1)=\pm p_1^2, \qquad N(\alpha)=\pm p_1.$$

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