AN INVARIANT OF A GENERAL TRANSFORMATION OF SURFACES*

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1. *Introduction.* If two surfaces, S and S', are in one-to-one point correspondence, the transformation T of S into S' establishes between the pencils of tangent lines at corresponding points of S and S' a projective correspondence. Furthermore, if the line of intersection, L, of the tangent planes to S and S' at the corresponding points M and M' passes through neither of these points, that is, if neither S nor S' is a focal surface of the congruence of lines MM', the pencils of tangent lines at M and M' cut L in projective ranges of points.

The invariant cross ratio of the projectivity on L is an invariant of the transformation T with respect to the group of collineations of the three-dimensional space in which S and S' are imbedded. We propose to study this invariant, and to apply it, in particular, to the so-called fundamental transformations of surfaces.

2. *General Case.* We shall restrict ourselves primarily to the general case in which the projective correspondence on L has two distinct fixed points, D_1 and D_2. Let the surfaces S and S' be represented parametrically so that corresponding points have the same curvilinear coordinates (u,v). In particular, take as the u-curves the corresponding families of curves on S and S' whose tangents at corresponding points, M and M', intersect in D_1 and, as the v-curves, the curves whose tangents at corresponding points intersect in D_2.

A. *Fixed Points Finite.* If D_1 and D_2 are both finite points expressions for their coordinates, $y^{(1)} : (y_1^{(1)}, y_2^{(1)}, y_3^{(1)})$ and $y^{(2)} : (y_1^{(2)}, y_2^{(2)}, y_3^{(2)})$ are readily found. Since, for example,

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