APPROXIMATE SOLUTIONS OF A SYSTEM OF DIFFERENTIAL EQUATIONS OF MATHEMATICAL PHYSICS BY LEAST SQUARES*

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1. *Systems of First Order.* In many problems of mathematical physics and particularly in electrical circuit theory, it is of importance to find approximate solutions of a system of differential equations of the form

$$(1)\qquad \frac{dx_i}{dt}=f_i(t,x_1,x_2,\cdots,x_p)\,,\qquad (i=1,2,\cdots,p)\,.$$

Sometimes it may be shown by physical considerations that a system of type (1) which corresponds to some definite experimental fact, really possesses a periodic solution with a period equal to T. By a suitable change of variables, we may suppose that

$$(2)\qquad x_i=0,\qquad (i=1,\cdots,p)\qquad \text{for}\qquad t=0,\qquad t=T;$$

and it then remains only to find the numerical solution of (1) and (2) with a given degree of approximation. We shall suppose first that the system (1) is linear with variable coefficients, that is to say, that

$$(3)\quad \frac{dx_i}{dt}-A_{i1}x_1-A_{i2}x_2-\cdots-A_{ip}x_p=F_i\,,\ (i=1,2,\cdots,p),$$

where A_{i1}, A_{i2}, $\cdots$, A_{ip}, F_i are functions of t. As in the method of least squares, we shall try to render stationary the integral

$$(4)\qquad \int_0^T\sum_{i=1}^p\left[\frac{dx_i}{dt}-\sum_{k=1}^p A_{ik}x_k-F_i\right]^2dt$$
$$=\int_0^T\sum_{i=1}^p\left[L_i(x_i)-F_i\right]^2dt$$

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