A THEOREM ON CONNECTED POINT SETS WHICH ARE CONNECTED IM KLEINEN*

BY R. L. WILDER

It has been shown by Miss Mullikin† that if K and M are two closed,‡ mutually exclusive point sets and H is a closed, bounded, connected set having at least one point in common with each of the sets K and M, then there exists a point set L, a subset of H, such that L is connected and contains no point of either K or M, but such that K and M each contain at least one limit point of L.

If H is not closed, the above theorem no longer holds, as can be shown by very simple examples. It is the purpose of this note to establish an analogous theorem for the case where H, although not closed, is connected im kleinen.§

THEOREM. *Let K and M be two closed mutually exclusive point sets and N a connected, connected im kleinen point set*

* Presented to the Society, October 31, 1925.

† *Certain theorems relating to plane connected point sets*, TRANSACTIONS OF THIS SOCIETY, vol. 24 (1922), pp. 144–162. Rosenthal gave a proof for that case where each of the sets K and M reduces to a single point. See A. Rosenthal, *Teilung der Ebene durch irreduzible Kontinua*, MÜNCHENER SITZUNGSBERICHTE, MATHEMATISCH-PHYSIKALISCHE KLASSE, 1919, p. 104.

‡ A point set is said to be (1) *closed*, if it contains all its limit points; (2) *connected*, if it is not the sum of two mutually exclusive points sets neither of which contains a limit point of the other; (3) *bounded*, if it lies entirely in a finite portion of the space under consideration.

§ A point set M is said to be *connected im kleinen* at a point P if for every circle K_1 with center at P there exists a concentric circle K_2 such that every point x of M which lies interior to K_2 is joined to P by a connected subset of M which lies wholly within K_1. M is itself said to be connected im kleinen if it is connected im kleinen at every point. See Hans Hahn, *Mengentheoretische Charakterisierung der stetigen Kurve*, WIENER SITZUNGSBERICHTE, vol. 123, Abt. IIa (1914), pp. 2433–2489; also *Über die allgemeinste ebene Punktmenge, die stetiges Bild einer Strecke ist*, JAHRESBERICHT DER VEREINIGUNG, vol. 23 (1914), pp. 318–322. See also S. Mazurkiewicz, *Sur les lignes de Jordan*, FUNDAMENTA MATHEMATICAE, vol. 1 (1920), pp. 166–209, and earlier papers in Polish referred to therein.