COVERING THEOREMS*

BY R. L. MOORE

A set of segments will be said to cover a point set F *in the Vitali sense* if, for every point P which belongs to F and every positive number e, there exists a segment of G which contains P and is of length less than e. J. Splawa-Neyman has shown† that if, in space of one dimension, F is a closed and bounded point set of measure zero and G is a set of segments which covers F in the Vitali sense, then, for every positive number e, the set G contains a subset G_e such that G_e covers F and such that the sum of the lengths of the segments of the set G_e is less than e. He cites the question, raised by Sierpinski, whether this theorem remains true after the removal of the condition that F be closed. I have recently‡ answered this question in the negative. Splawa-Neyman shows, by an example, that his theorem does not hold true for two dimensions, but makes the following statement, without proof:

"Remarquons que notre théorème subsiste pour les espaces à n dimensions, s'il existe pour tout point p de F une sphère appartenant à F de rayon aussi petit que l'on veut et *dont le centre est en p*."

In the present paper I will show that the theorem thus stated, without proof, by Splawa-Neyman remains true on the removal of the condition that F be closed. I will also show that the condition that each point of F be the center of spheres of

* Presented to the Society, in a somewhat different form, December 30, 1924. In the abstract of this paper printed in this BULLETIN, vol. 31 (1925), pp. 219-220, proposition (2) is not correctly worded and, as will be shown below, (3) is false.

† *Sur un théorème métrique concernant les ensembles fermés*, FUNDAMENTA MATHEMATICAE, vol. 5 (1924), pp. 328-330.

‡ Cf. R. L. Moore, *Concerning sets of segments which cover a point set in the Vitali sense*, PROCEEDINGS OF THE NATIONAL ACADEMY, vol. 10 (1924), pp. 464-467.