NOTE ON A PROBLEM IN APPROXIMATION WITH AUXILIARY CONDITIONS*

BY DUNHAM JACKSON

Let $\rho(x)$ and $f(x)$ be two given functions of period 2π, the former bounded and measurable, with a positive lower bound, the latter, for simplicity, continuous. Among all trigonometric sums $T_n(x)$, of given order n, there is one and just one for which the value of the integral

$$(1) \qquad \int_0^{2\pi} \rho(x)\,[f(x)-T_n(x)]^2 dx$$

is a minimum. If the weight function $\rho(x)$ is identically 1; it is a matter of familiar knowledge that the minimum is reached when $T_n(x)$ is the partial sum of the Fourier series for $f(x)$. A considerable amount of attention has been given recently to the problem of the convergence of the minimizing sum $T_n(x)$ toward $f(x)$, as n becomes infinite, under the generalized conditions that result from the admission of an arbitrary weight function.†

Let $x_1, \cdots, x_N$ be N values of x in the interval $0 \leqq x < 2\pi$. The problems of the preceding paragraph may be further varied by admitting to consideration only such sums $T_n(x)$ as satisfy the conditions

$$(2) \qquad T_n(x_i) = f(x_i), \qquad (i = 1, 2, \cdots, N),$$

and inquiring after the minimum of the integral (1) subject to these auxiliary conditions. It is understood that the given value of n is large enough so that the conditions (2) can be

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† Cf. e.g., D. Jackson, *Note on the convergence of weighted trigonometric series*, this BULLETIN, vol. 29 (1923), pp. 259–263, where further bibliographical references will be found; also D. Jackson, *A generalized problem in weighted approximation*, TRANSACTIONS OF THIS SOCIETY, vol. 26 (1924), pp. 133–154.