

thesis that the theorem is false for I , and establishes the desired result.

The above theorem may be stated otherwise as follows. If the sequence of horizontal functions $h(x, n)$ satisfying the conditions of the lemma approach the zero limit function monotonically, they must approach it uniformly. It follows as a corollary from this theorem that if we have a series of "interval" functions, such that $f(x, n)$ is a single-valued continuous curve in each subinterval, and if the sequence of functions approach a continuous limit function monotonically for each fixed x in I , then the approach to the limit function is uniform with respect to x . In particular, the sequence of "interval" functions may be a set of functions $f_n(x)$ each of which is continuous in I .

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NOTE ON RATIONAL PLANE CUBICS*

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1. *Introduction.* Many constructions have been devised for a rational plane cubic. One of the most interesting of them is due to Zahradnik[†] who noticed that the familiar construction for the cissoid of Diocles could be extended so as to generate any rational plane cubic. It is as follows: Take any conic C , a fixed point O on C , and a fixed line b . Any line l through O meets C a second time at P , and b at Q . On l lay off a segment OM equal to and in the sense PQ . The locus of the point M is a rational plane cubic R with double point at O . The tangents to R and O are the joins of O to the two points in which b meets C .

Niewenglowski[‡] showed a bit later that this same construction may be applied to R , using a second fixed line dis-

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† Zahradnik, *Cissoïdcurven*, ARCHIV DER MATHEMATIK UND PHYSIK, vol. 56, p. 8.

‡ Niewenglowski, *Sur les courbes d'ordre n à point multiple d'ordre $n-1$* , COMPTES RENDUS, vol. 80 (1875), p. 1067.