THE HEAVISIDE OPERATIONAL CALCULUS*

BY J. R. CARSON

The Heaviside operational calculus is a systematic method, originated and developed by Oliver Heaviside, for the solution of systems of linear differential equations with constant coefficients, and linear partial differential equations of the type of the *wave equation*. Its important applications in mathematical physics are to the dynamics of small oscillations, the Fourier theory of heat flow and to electrical transmission theory. It was, in fact, in connection with problems of the latter class that the operational calculus was developed, and in the solution of such problems it is an instrument of remarkable directness, simplicity and power.†

The operational calculus may be described with sufficient generality for our present purposes, in connection with the solution of the system of linear differential equations

$$
(1) \qquad \left\{
\begin{array}{l}
a_{11}x_1+a_{12}x_2+a_{13}x_3+\cdots+a_{1n}x_n=f_1(t)\ , \\
a_{21}x_1+a_{22}x_2+a_{23}x_3+\cdots+a_{2n}x_n=f_2(t)\ , \\
\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\
a_{n1}x_1+a_{n2}x_2+a_{n3}x_3+\cdots+a_{nn}x_n=f_n(t)\ ,
\end{array}
\right.
$$

where the coefficient a_{jk} has the form

$$
(2) \qquad a_{jk}=\alpha_{jk}+\beta_{jk}\frac{d}{dt}+\gamma_{jk}\frac{d^2}{dt^2}+\cdots .
$$

$\alpha, \beta, \gamma, \cdots$ being constants. We require the solution for $x_1, x_2, \cdots, x_n$ for the following boundary conditions: The known functions $f_1, \cdots, f_n$ and the dependent variables $x_1, \cdots, x_n$ are identically zero for $t<0$. In the language of dynamics, the "forces" $f_1, \cdots, f_n$ are applied to a system

* An address delivered before the Society, May 2, 1925.

† For a brief discussion of the Heaviside operational calculus from the point of view adopted in this paper, see Carson, *The Heaviside Operational Calculus*, BELL SYSTEM TECHNICAL JOURNAL, Nov., 1922.