

*L'Analysis Situs et la Géométrie Algébrique.* By S. Lefschetz. Paris, Gauthier-Villars, 1924.

As a condensed account of the theory of algebraic manifolds, this little volume is so successful that it would seem beside the point to criticise the somewhat hasty manner in which the author disposes of several rather delicate details. Professor Lefschetz has not only given the theory a new unity, but has clarified and extended it at more than one point by the systematic use of topological methods.

First, we are given a brief résumé dealing with the theory of cycles on an algebraic surface, the connectivity numbers and coefficients of torsion, the intersection numbers associated with cycles intersecting in points. Skilful use is made throughout the book of the simple theorem that if one of two cycles intersecting in points is a bounding cycle, the algebraic sum of the points of intersection is zero. It may be mentioned here, in passing, that there are a number of topological invariants associated with the theory of cycles that meet, not in points, but in curves, surfaces, etc. So far as the reviewer knows, the significance of these invariants in the theory of algebraic manifolds has never been studied.

There follows a discussion of the topological characteristics of an algebraic surface, during the course of which a number of the fundamental constants of algebraic geometry reappear as topological invariants. The surface is thought of as swept out by its intersection with a plane varying in a pencil of planes. A general plane section of the surface is a curve  $C$  of genus  $p$  on which  $2p$  1-cycles,  $\gamma_1, \gamma_2, \dots, \gamma_{2p}$ , independent with respect to  $C$ , may be traced. If the plane is made to vary in its pencil so as, ultimately, to return to its original position, the cycles  $\gamma_i$  undergo a linear transformation. It would be interesting to see how far the theory of algebraic manifolds could be carried without making use of anything more than the group of linear transformations engendered on the cycles  $\gamma_i$ . Along these lines, it might be possible to develop a perfectly general theory of surfaces without going into the question of the resolution of singular points and curves.

There is an important theorem due to Lefschetz himself to the effect that a cycle is algebraic if and only if no double integrals of the first kind have periods with respect to it. This and other considerations suggest that, in the theory of algebraic manifolds, it might be advisable to modify the definition of an homology so as to make it read as follows: A cycle  $C$  is homologous to zero,  $C \sim 0$ , if, and only if, it is bounding or algebraic. The invariants (Betti numbers, coefficients of torsion, etc.) that would follow from this definition of an homology would have the advantage of being unaltered under birational transformations, whereas the strictly topological invariants that follow from the ordinary definition do not have this property.