CHARACTERISTIC PARAMETER VALUES FOR AN INTEGRAL EQUATION*

BY W. A. HURWITZ

1. *Introduction; Hermitian Kernel.* It is well known, though seldom explicitly mentioned, that the properties of real symmetric kernels hold, with only trivial alterations in the usual proofs, for complex Hermitian kernels. It is the purpose of this note to show that kernels of certain other kinds have analogous properties, with the role of the real axis as bearer of the characteristic parameter values taken by any other straight line in the plane.

All functions of x, y, s which appear will be understood to be complex functions defined for real values of each of the variables in the closed interval (a, b); all integrals will be taken between the limits a, b, which will not be written. We call λ a *characteristic parameter value* (hereafter abbreviated cpv) for the kernel $K(x, y)$ if there exist non-trivial solutions of the equations

$$u(x) = \lambda\int K(x, s)u(s)ds, \tag{1}$$

$$v(y) = \lambda\int v(s)K(s, y)ds. \tag{2}$$

The cpv's and the corresponding solutions of (1) (normalized and orthogonalized), arranged in the usual order, will sometimes be denoted by $\lambda_p, \varphi_p(x)$ $[p = 1, 2, \cdots]$. A cpv is a pole (for at least some values of x, y) of the resolvent function $K(x, y; \lambda)$, which is elsewhere analytic in λ. The resolvent satisfies the equation

$$K(x, y; \lambda) - K(x, y; \mu) = (\lambda - \mu)\int K(x, s; \lambda)K(s, y; \mu)ds \tag{3}$$

whenever neither λ nor μ is a cpv. Since $K(x, y; 0) = K(x, y)$, (3) reduces for $\lambda = 0$ and for $\mu = 0$ to the simpler resolvent formulas used in the process of solving the Fredholm equation.

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