CRITERIA THAT ANY NUMBER OF REAL POINTS IN n-SPACE SHALL LIE IN AN $(n-k)$-SPACE

BY H. S. UHLER

The object of the present paper is to establish an algebraic identity from which may be deduced necessary and sufficient conditions that any large number of real points in n-dimensional linear space shall lie in a linear $(n-k)$-space.

Let the following matrix, in which the number of columns is m and the number of rows is $n+1\,[m \geqq (n+1)]$, be compounded with its conjugate:

$$\begin{matrix} 1 & 1 & \cdots & 1 \\ x_{1,1} & x_{2,1} & \cdots & x_{m,1} \\ x_{1,2} & x_{2,2} & \cdots & x_{m,2} \\ \cdot & \cdot & \cdots & \cdot \\ x_{1,n} & x_{2,n} & \cdots & x_{m,n}. \end{matrix}$$

The determinant of the resulting symmetric square array is

$$\begin{vmatrix} m & \Sigma x_{i,1} & \Sigma x_{i,2} & \cdots & \Sigma x_{i,n} \\ \Sigma x_{i,1} & \Sigma x_{i,1}x_{i,1} & \Sigma x_{i,1}x_{i,2} & \cdots & \Sigma x_{i,1}x_{i,n} \\ \cdot & \cdot & \cdot & \cdots & \cdot \\ \Sigma x_{i,n} & \Sigma x_{i,n}x_{i,1} & \Sigma x_{i,n}x_{i,2} & \cdots & \Sigma x_{i,n}x_{i,n} \end{vmatrix} \equiv \Delta;$$

$$(i = 1, 2, 3, \cdots, m).$$

Multiply all of the rows of Δ except the top row by m, compensate by prefixing m^{-n}, and remove the factor m now common to the constituents of the first column to get

$$\Delta = m^{1-n}\begin{vmatrix} 1 & \Sigma x_{i,1} & \Sigma x_{i,2} & \cdots & \Sigma x_{i,n} \\ \Sigma x_{i,1} & m\Sigma x_{i,1}x_{i,1} & m\Sigma x_{i,1}x_{i,2} & \cdots & m\Sigma x_{i,1}x_{i,n} \\ \cdot & \cdot & \cdot & \cdots & \cdot \\ \Sigma x_{i,n} & m\Sigma x_{i,n}x_{i,1} & m\Sigma x_{i,n}x_{i,2} & \cdots & m\Sigma x_{i,n}x_{i,n} \end{vmatrix};$$

$$(i = 1, 2, 3, \cdots, m).$$