ON THE NUMBER OF ELEMENTS OF A GROUP WHICH HAVE A POWER IN A GIVEN CONJUGATE SET*

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1. *Introduction.* A fundamental theorem on abstract groups is Frobenius' theorem: The number of elements in a group of order g whose nth powers belong to a given conjugate set is zero or a multiple of the greatest common divisor of g and n. In this paper, I will prove the following theorems, which are also concerned with the number of elements having a power in a given conjugate set.

THEOREM 1. *The number of elements of a group whose nth powers are in a given conjugate set is either zero, or a multiple of the number of elements in the conjugate set.*

THEOREM 2. *In a group of order g, the number of elements which have a power in a given conjugate set of elements of order n is a multiple of the greatest divisor of g that is prime to n.*

An interesting deduction from Theorem 2 is the following theorem.

THEOREM 3. *In a group of order g, the number of elements whose orders are multiples of n is either zero, or a multiple of the greatest divisor of g that is prime to n.*

2. *Proof of Theorem* 1. Let $t_1, t_2, \cdots, t_x$ be the elements of a group G which satisfy the equation $t^n = s_1$, and let the conjugates of s_1 under G be $s_1, s_2, \cdots, s_m$. There exist elements $u_1, u_2, \cdots, u_m$ in G such that

$$u_i^{-1} s u_i = s_i, \qquad (i = 1, 2, \cdots, m).$$

Since $t_a^n = s_1$,

$$(u_i^{-1} t_a u_i)^n = u_i^{-1} s u = s_i.$$

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