A HISTORICAL NOTE ON GIBBS' PHENOMENON IN FOURIER'S SERIES AND INTEGRALS

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In 1899, Gibbs called attention* to the fact that for large values of n the approximation curves

$$y = S_n(x) = 2\sum_1^n (-1)^{r-1} \frac{\sin rx}{r},$$

for the Fourier's series which represents $f(x) = x$ in the interval $-\pi < x < \pi$, fall from the point $(-\pi, 0)$ at a steep gradient to a point very nearly at a depth $2\int_0^\pi[(\sin \alpha)/\alpha]d\alpha$ below the axis of x, then oscillate above and below $y = x$ close to this line until x approaches π, when they rise to a point very nearly at a height $2 \int_0^\pi[(\sin \alpha)/\alpha]d\alpha$ above the axis, and then fall rapidly to $(\pi, 0)$.

At the point of discontinuity, where $x = \pi$, in the series $2\sum_1^\infty(-1)^{r-1}(\sin rx)/r$ the approximation curves thus tend to coincide, not with the segment joining the points (π, π) and $(\pi, -\pi)$, but with the straight line whose ends are the points

$$\left(\pi, \pi + \frac{D}{\pi}\int_\pi^\infty \frac{\sin \alpha}{\alpha}\, d\alpha\right)$$

and

$$\left(\pi, -\pi - \frac{D}{\pi}\int_\pi^\infty \frac{\sin \alpha}{\alpha}\, d\alpha\right),$$

where $D = f(\pi + 0) - f(\pi - 0)$, the amount of the "jump" in the sum of the series at that point.

In 1906, Bôcher showed† that the same phenomenon occurred in general in the Fourier's Series for the arbitrary

* NATURE, vol. 59 (1899), p. 606.

† ANNALS OF MATHEMATICS, (2), vol. 7, (1906).